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PHD. THESIS

SUMMARY

Advanced Nonlinear Analysis of Frames Composed of Tapered Members And Flexible Connections

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2016

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Introduction

In recent decades, due to economic and social changes, an exponential growth of the cities can be seen, due to population migration. With limited space, the architects had to seek new solutions, thus building on "vertical" had been made possible. With the evolution of modern architectural style, creating a simple and aesthetic design that allows efficient use of functional space was one of the main concerns of the architects. Hence, extremely durable materials such as steel allow building structures with unconventional shapes. Also, the steel structures are becoming more accessible due to its easy assembly, the short construction period and reasonable costs. For these reasons, the design of steel structures has grown significantly in recent years and it is an affordable solution for both industrial and modern buildings, Fig. **Error! No text of specified style in document.**.1.

*Fig. **Error! No text of specified style in document.**.1 Steel Structures: (a) Eiffel Tower (Wikipedia), (b) Burj Khalifa (Wikipedia),*
(c) Industrial buildings, (d) National Stadion fromBeijing

The necessity and opportunity of the reseach theme

Conventional structural design methods provide prescriptions based on individual element capacity checks, but are limited in terms of assessing the overall structural behaviour, taking into account the interaction between the elements. This aspect is extremely important for complex structures, where an incorrect estimation of the ultimate resistance capacity of the entire structural system can lead to disadvantaged solutions economically or structurally. Also, considering that conventional design methods are not able to trace the complete load-displacement curve, failure mechanism cannot be rigorously surprised and, for steel structures with slender elements, failure can occur due to stability loss and not to loss of load bearing capacity.

With rapid development of computer technology and research progresses, FEM based software became very popular and can reflect with a high degree of precision the real behaviour of structures, but they are limited in the analysis of complex structures, requiring very fine-grained modelling and extensive calibration that are often impractical to structural engineer and implies a high computational effort. For this reason, finite element analysis based methods are used in research, for the calibration of other simplified analysis models (ECCS, 1984), (White & Chen, 1990).

Currently the need of new methods that allow a global analysis of the structures, without the need of any additional individual element capacity checks, is well known. Hence, researchers have been concerned in developing new advanced analysis methods for facilitating the evaluation of response of structures in current practice. There are several analytical methods, for the second order inelastic analysis of steel structures mentioned in the literature, which can be classified in two main categories, namely: plastic hinge based concept and plastic zone model respectively. An analysis method is considered "advanced" if it can sufficiently describe the strength, stiffness and overall stability of the structure, such that separate member capacity checks are not required (Chen W. , 1993), (Li & Li, 2007), (Maleck, White, & Chen, 1995). In order to correctly evaluate the real behaviour of the structure, such method must be able to capture the effects of local and global geometric nonlinearity, material nonlinearity (considering plastic interaction curves), the initial geometric and mechanical imperfections, bowing effect and flexible connections behaviour.

After going through existing research literature on advanced analytical methods able to engage the main sources of nonlinearity and imperfections, the following remarks that highlight some limitations (drawbacks) of existing methods, which lead to the need of improving these methods for the second order inelastic analysis of steel frame structures composed of prismatic or tapered elements:

- Although some analysis methods mentioned in the literature are able to identify the plastic failure mechanism, for steel structures with slender members subjected to axial and lateral loads, failure mechanism may occur due to global instability due to the emergence of second order bending moments caused by the action of axial compression forces on the deformed shape. In these situations it is recommended to express the equilibrium equations on the deformed configuration and, at element level, coupling physical and geometric nonlinearities is, generally, difficult to achieve when using a single element per member.

- Including elements that are subjected to distributed loads or concentrated loads along the element length in global analysis is required in daily practice. Thus, in order to capture plastic hinge formation along the member length, several approaches are proposed, but most of them need to divide the bar into several segments / elements (Chen & Chan, 1995), (Wong, 1996), (Kim, Lee, Choi, & Kim, 2004), which implies a higher computational effort due to additional data that must be stored, as a result of the reconfiguration of the geometry of the structure (topology change).

- In the classic plastic hinge approach, the effect of material yielding is ‘‘lumped’’ into a dimensionless plastic hinge. Regions in the frame elements other than at the plastic hinges are assumed to behave elastically, and if the cross-section forces are less than cross-section plastic capacity, elastic behaviour is assumed. Unfortunately, as plastification in the member is assumed to be concentrated and occurs instantaneously, this method can lead to an overestimation of the element stiffness. For this reason, it is recommended to use an analytical model that is able to capture the gradual stiffness degradation of cross-sections and, with some degree of precision, gradual spread of yielding throughout the volume of the members.

- In reality, regardless of the quality of the manufacturing process, steel profiles develop imperfections, geometrical or mechanical, which could affect the stability of the structure and may cause premature collapse. Very early experimental research showed large discrepancies between theoretical and experimental results and, thus, confirms the importance of including the effects of initial imperfections in the analysis model (Wilson & Brown, 1935), (Koiter, 1945). In order to obtain a structural response closer to reality, the effect of these imperfections on the behaviour of steel frames, has been included, in a simple manner, by using the tangent modulus Et in stiffness matrix, but this approach is known to overestimate the ultimate strength capacity of the structure (Maleck A. , 2001). Therefore, is necessary to develop new techniques that include more rigorously the effect of initial imperfections.

- With rapid evolution of modern architectural style, steel structures allow using unconventional shapes. Hence, research efforts to study the behaviour of elements with variable section have grown exponentially in recent years. Several methods are mentioned in the literature, but some of these do not include the effect of axial forces (Cleghorn WL, 1992), (Frieman Z, 1992), (To, 1981) in expressing equilibrium relations at element level (in stiffness matrix component). Also, most of these methods do not take into account the effects of distributed loads, even if their importance in the current design is well known (Li & Li, 2002), (Al-Sadder, 2004).

Although several advanced models for the nonlinear analysis of tapered steel frames have been developed, some of them mentioned above, several important features for practical applications, the combined effects of element geometrical nonlinearity, initial geometric imperfections, lateral loads applied on member length, shear deformation, semi-rigid behaviour of the connections and the ability to capture plastic hinge development inside the element length by using only one line element per structural member are not completely and efficiently developed in the computational methods addressed in the literature.

In this context, it is believed necessary and appropriate to carry further research for the nonlinear advanced static analysis of frame structures composed of tapered members and flexible connections.

The main objectives of the thesis

Based on the existing literature review, in order to overcome the inconveniences mentioned above, the current research has three main objectives:

- **The main objective is to develop an advanced analysis** model able to capture, with a high degree of accuracy and low computational effort, the nonlinear behaviour of **steel structures with tapered elements and semi-rigid connections**. In this context, the present work attempts to develop a tapered element able to capture the combined effects of second-order geometrical nonlinearities, material nonlinearities, shear deformation, initial geometrical and mechanical imperfections and semi-rigid connections. Hence, to solve the second order differential equilibrium equation, with the rigid body modes removed, and considering as a primary unknown the bending moment, the power series approach is used and, by combining with the Maxwell-Mohr method, the force-displacement relationship at the element level is computed, highlighting in this way the flexibility matrix and equivalent nodal loads of the tapered element.

- Subsidiary objective of present research is to propose and calibrate **a practical procedure for the evaluation of the stiffness matrix and equivalent nodal forces vector for tapered members**. The practical character of the proposed approach consists in integrating the bending moment expression for prismatic elements, in evaluating the flexibility coefficients using Maxwell-Mohr method, considering, however, in an approximately form, the variation of the moment of inertia along the element length. Hence, through all extensive calibration studies that have been made, a medium compression coefficient is determined which takes into account the end depth ratio. By inverting the incremental displacement-force relations, the stiffness matrix and equivalent nodal forces vector are highlighted. This procedure is effective in daily design of structures, and it is shown that can be applied in most practical cases; the errors introduced are, generally, below 5%.

- In order to validate the theoretical model developed herein, **a software application has been developed in Matlab**, for the nonlinear advanced analysis of steel structures with tapered elements and semi-rigid connections. A plastic hinge method is adopted to simulate member plasticity, using a single element per member; plastic hinges are assumed to be lumped either at the ends of the beam-column element or along the element length. The concentrated plasticity model proposed in (Chiorean C. G., 2009) for the derivation of force-displacement relation **is extended** for non-prismatic elements in conjunction with consideration of second order effects and lateral loads in the bending moment expression and initial geometric imperfections. The effect of semi-rigid connections is included in the second-order plastic hinge analysis using the zero length rotational spring element approach. To simulate the spread of plasticity across cross-sections and throughout the volume of the members, the formulation proposed by Zubydan (2010) is used, for evaluating the tangent modulus of elasticity, considering three sections susceptible to plastification, the ends of the element and along its length.

The efficiency, accuracy and robustness of the analytic procedure and the computer program developed here has been evaluated using **several benchmark problems analysed** previously by other researchers using independent analytical, numerical and finite element solutions. These studies aim, on one hand, to validate the analytical formulation, and, on the other hand, to provide an information base for future studies of other researchers in this field.

Thesis outline

The present work contains six chapters and five appendices, a brief description of each is presented below:

Chapter 1. Introduction

This chapter begins with the importance of steel structures, which has become a more accessible solution due to its easy assembly, the short construction period and reasonable costs. Then, it covers a review of current literature relevant to present work, highlighting some inconveniences of existing advanced analysis methods and the main objectives of present research. Research methodology underlying present work has also been briefly described.

Chapter 2. Second order elasto-plastic analysis methods of frame structures – literature review

First part of Chapter 2 presents existing advanced analysis methods of frame structures, which can be categorized in two main types, plastic hinge versus spread of plasticity approaches. In addition, it provides information about the main sources of nonlinearity and various techniques mentioned in the literature for including them in the analysis model. At the end of the chapter, some techniques are briefly described for the numerical solution of the equilibrium equations associated with nonlinear analysis; two different mathematical approaches will be covered, namely single-step and interactive incremental methods.

Chapter 3. Geometric nonlinear analysis of tapered elements

Chapter 3 starts with a review of current literature relevant to analysis of tapered elements. Research work focused on existing techniques based on power series approach and the highlighted drawbacks justifies the author's motivation on exploring new approaches. Following the limitations mentioned above, two mathematical models are presented herein. The first one implies integrating the bending moment expression for prismatic elements, in evaluating the flexibility coefficients using Maxwell-Mohr method, considering, in an approximately form, the variation of moment of inertia along the element length. In this way, by inverting the incremental displacement-force relations, the stiffness matrix and equivalent nodal forces vector are highlighted. This procedure is effective in daily design of structures, and it is shown that if end section depth ratio is less than 0.6 the errors introduced are, generally, below 5%. The second proposed approach, which is considered "exact" in theoretical formulation, combines the power series approach, used for solving the second-order differential equation of equilibrium, with the Maxwell-Mohr method to compute the flexibility coefficients of the Euler-Bernoulli beam-column element in the presence of initial geometric imperfections, axial load and distributed loads acting along the element length. The originality and effectiveness of the proposed method consists in solving the second order differential equilibrium equation, with variable coefficients, for an element with rigid body modes removed, considering as a primary unknown the bending moment, by applying the power series approach. The proposed governing equation and the resulted element stiffness matrix and equivalent nodal load vector are of general forms, they can be applied to any variation of sectional shape along member length subjected to uniform axial compressive or tensile forces.

At the end of this chapter, in order to verify the effectiveness and accuracy of the proposed procedures and the computer program developed herein , numerical studies regarding the convergence of integration studied methods and the appropriate number of terms in the power series used in expressing bending moment have been performed.

Chapter 4. The proposed method for the advanced analysis of frame structures. Software application EPASS.

Chapter 4 describes the analytical procedure, covering the nonlinear geometrical (local and global) and physical effects, initial geometrical and mechanical imperfections, the second order effects, shear deformations effects as well as the flexible connections, underlying the developed software application, EPASS, in Matlab 7.11. The proposed formulation is intended to model the geometrically nonlinear inelastic behaviour of composite elements using only one element per physical member which reduces the number of degree of freedom involved and the computational time. A plastic hinge method is adopted to simulate member plasticity; plastic hinges are assumed to be lumped either at the ends of the beam-column element or along the element length, regions in the frame elements other than at the plastic hinges are assumed to behave elastically. In the present work the concentrated plasticity model proposed in (Chiorean C. G., 2009) for the derivation of force-displacement relation is extended for non-prismatic elements in conjunction with consideration of second order effects and lateral loads in the bending moment expression and initial geometric imperfections. The method also ensures that the plastic bending moment is nowhere exceeded along the member length once a plastic hinge develops without the need to divide the member into two elements; a procedure is used to enforce the force-point movement to remain on the plastic strength surface of a member, once a full plastified section develops. In order to simulate the spread of plasticity across cross-sections and throughout the volume of the members, the formulation proposed by Zubydan (2010) is used. Distributed lateral loads acting along the member length can be directly input into the analysis without the need to divide the member into several elements leading to a consistency in the linear and the nonlinear structural models. The elastic unloading and strain-hardening effects are not included in analysis.

The analysis algorithm is also detailed; for the equilibrium path to be traced, an incremental-iterative method with force control has been used. The accuracy of the analytic procedure and the computer program developed here has been extensively tested through comparisons with the established results available from the literature and the obtained results prove the effectiveness of the proposed method.

Chapter 5. Numerical examples for validation and calibration of the developed software program for nonlinear analysis of frame structures

The aim of chapter 5 is to provide enough results to prove the efficiency, accuracy and robustness of the analytic procedure and the computer program developed, EPASS. Several benchmark problems analysed previously by other researchers using analytical (McGuire, Gallagher, & Ziemian, 2000), numerical and finite element solutions (Hibbitt, 2011) have been studied. The selected problems consist in both isolated elements and two dimensional frames that are sensitive to different nonlinear effects that have been included in the analysis. All the performed analysis are nonlinear elastic and elasto-plastic static analysis, local stability loss, both lateral and torsional buckling, have been neglected in the model.

The numerical results presented are relevant and confirms the performance of the analytical model and ranks it as an advanced static nonlinear analysis method for planar steel frame structures.

Chapter 6. Conclusions, personal contributions and future research directions

Chapter 6 presents general conclusions, original contributions of conducted research are, also, covered. The proposed approach herein, for the nonlinear static analysis of frames composed of tapered elements, intents to overcome some of the inconveniences of existing methods mentioned in the literature. Extensive numerical studies, that have been conducted in present work, aim to expand currently knowledge and, also, to establish a useful database for researchers in their attempt of calibrating different applications regarding nonlinear static analysis of steel frames. Subsequently, future research directions are briefly presented.

Ultimately, the consulted bibliography is presented, along with the five appendices that contain: Appendix A: stability functions proposed by Oran (1973) and Chan & Gu (2000) for including the local geometric nonlinear effect in stiffness matrix of the element; Appendix B: stiffness matrix analytical expressions deducted with Matlab 7.11, for a beam-column element with plastic hinges taking into account the second order effects; Appendix C: analytical expressions proposed by Zubydan (2010) to evaluate the coefficients that simulate gradual plastification of the element, that are included in the stiffness matrix; Appendix D: a summarized description of integration methods Simpson 3/8, Gauss-Legendre and Gauss-Lobatto quadrature and Appendix E: stability functions for solid circular tapered beam–column element in comparison with Al-Sadder (2004) results.

General conclusions

With rapid development of computer technology, advanced analysis of structures has become feasible. Hence, advanced analysis methods can describe, with some degree of accuracy, strength, stiffness and overall stability of the structure, so that additional member checks are no longer needed (Chen W. , 1993) (Maleck, White, & Chen, 1995), (Li & Li, 2007), (Chiorean C. G., 2009). The failure mechanism can, also, be captured by stability loss, bearing capacity loss or their combined effect. Detecting the positions and the formation order of plastic hinges allow to capture internal forces redistribution leading to a more economical design of structures, reveals ductility capacities and give a higher degree of safety.

In recent years, seismic or exceptional action design of building structures has faced an obvious clarification and adjusting process to the latest observations, somehow surprising, provided by the major earthquakes in last years and, also by progressive collapse phenomenon caused by explosions or terrorist attacks. New answers have been found to the questions relating performance based structural design – force based design or displacement based design, elastic or inelastic structural behaviour evaluation, linear or nonlinear static or dynamic analysis, deterministic or probabilistic analysis, etc. Design codes seek to provide better control design to satisfy the strength, ductility and deformation control requirements for structures subjected to both major actions, but rare, and some moderate or minor actions, but with higher frequency of occurrence. On the other hand, despite the spectacular progresses that computer technology has made in recent years, static or dynamic nonlinear analysis continuously demand prohibitive execution time, especially when fiber element based software application are used (spread of plasticity) in order to model the nonlinear response of high structures with irregularities in plan and elevation which, usually, require tridimensional models composed of hundreds or even thousands of elements.

Nonlinear static analysis, despite the fact that simplifies the dynamic character of the action, provides important information regarding the structural response, for example: (1) identifies critical zones where significant inelastic deformations may occurs, (2) highlights strength irregularities, in plan and elevation, which could produce considerable changes of static or dynamic inelastic response, (3) strength requirements evaluation for elements susceptible to brittle fracture and (4) prediction of plastic hinge formation sequence and stability failure of the elements.

The above considerations constitutes a sufficient reason for that static nonlinear analysis to continue to be refined, so that these techniques can become safer and more useful, whether they are used to replace analytical nonlinear dynamic analysis methods of relatively simple structures in seismic design, or if they are complementary used to dynamic analysis for complex structures. It should, also, be mentioned that the topic discussed in this paper is part of recent, or less recent, concerns of the members of Department of Structural Mechanics (MECON), from Technical University of Cluj Napoca. Developing new nonlinear or stability analysis methods for prismatic or non-prismatic elements is a complex, traditional and an actuality topic, which is justified by the number of prestigious scientific papers of the members of this department, mentioning only some of them (Alexa, 1976), (Barsan, 1978), (Pantel, 1976), (Chisalita, 1983), (Kopenetz ,1989), (Chiorean C. G., 2001), (Popa, 2003), (Nedelcu, 2009), (Chira, 1999), (Bors, 2014).

The proposed objectives of the conducted research fit directly into this national and international trend of developing and improving advanced nonlinear static analysis with direct impact on design codes and, implicitly, on daily structural design. Hence, the main objectives and the level of accomplishment will be summarized in the following.

The present work attempts to develop accurate yet computational efficient tools for the nonlinear inelastic analysis of steel frames with tapered members and semi-rigid connections. Hence, in order to evaluate the nonlinear response of steel structures, considering the second order local and global nonlinear geometric effects, initial geometric or mechanical imperfections, shear deformation effects and the effects of flexible connections of the elements, a software application has been developed, EPASS, in Matlab 7.11. The analysis is incremental-iterative with force control, snap-back and snap-thru phenomena, as well as post critical behaviour are not possible.

In order to compute the force-displacement relationship at the element level the Maxwell-Mohr relations are used, highlighting in this way the flexibility matrix and equivalent nodal loads of the element. Such a formulation allow us to take into account in a more efficient manner the initial geometrical imperfections and member lateral loads, the effects of shear deformation are integrated directly in force-displacement relationships by means of applying the Maxwell-Mohr rule to compute the generalized displacement in the second-order geometrically nonlinear analysis. In this respect the element force fields are described by the second-order bending moments and shear forces derived by solving the second-order differential equation expressing the variation of the bending moment along the member length in the presence of the axial force, member lateral loads and the second-order effects associated with the initial geometric imperfections. In this way the elements of the stiffness matrix and equivalent nodal loads can be obtained analytically and readily evaluated by computing the *correction coefficients* that affect the first-order elastic flexibility coefficients and equivalent nodal loads. Besides, the effect of the transverse shear deformation can be readily included in the element formulation, both in stiffness matrix and equivalent nodal loads. We note that for non-prismatic elements, in geometric nonlinear analysis, the bending moment expression along the element length can no longer be evaluated accurately, since the second order differential equation becomes nonlinear with variable coefficients (EI flexural rigidity varies along the member) and an analytical solution of this equation is generally difficult to obtain, in some cases even impossible. Therefore, it has been considered opportune to pursue other methods in order to determine the bending moment expression in geometric nonlinear analysis (including the effect of axial force, compression/tension, in equilibrium equation expressions at element level). The first proposed method is a practical technique which implies integration of prismatic elements with an equivalent flexural rigidity EI and applying the Maxwell-Mohr rule to compute the generalized displacement. In this way, in the bending moment expression a medium compression coefficient (equivalent) is introduced, to account the second order effects, which is evaluated taking into account the end sections inertia moment ratio. The second proposed method, considered herein as “exact”, is based on power series approach in solving the second order differential equilibrium equation with variable coefficients, with the rigid body modes removed, considering as a primary unknown the bending moment and, by combining with the Maxwell-Mohr method, the force-displacement relationship at the element level is computed, highlighting in this way the flexibility matrix and equivalent nodal loads of the tapered element. The proposed governing equation and the resulted element stiffness matrix and equivalent nodal load vector are of general forms, they can be applied to any variation of sectional shape along member length subjected to uniform axial compressive or tensile forces.

The nonlinear inelastic analysis employed herein takes the advantage of using only one beam-column element per physical member simultaneously considering the effects of second-order geometrical nonlinearities, shear deformation and initial geometric imperfections featuring in this way the ability to be used for practical applications by combining modelling benefits, computational efficiency and reasonable accuracy. Distributed lateral loads acting along the member length can be directly input into the analysis without the need to divide the member into several elements leading to a consistency in the linear and the nonlinear structural models. A plastic hinge method is adopted to simulate member plasticity; plastic hinges are assumed to be lumped either at the ends of the beam-column element or along the element length. The method ensures also that the plastic bending moment is nowhere exceeded along the member length once a plastic hinge develops without the need to divide the member into two elements and applying the plastic flow rules at the element ends as in (Orbison, McGuire, & Abel, 1982) or to make additional operations of static condensation as in (Liu, Liu, & Chan, 2014). In the present paper the concentrated plasticity model proposed in (Chiorean C. G., 2009) for the derivation of force-displacement relation is extended for non-prismatic elements in conjunction with consideration of second order effects and lateral loads in the bending moment expression and initial geometric imperfections.

The local geometric nonlinear effects are included in the analysis by updating, at every incremental load step, the geometric coordinates of the nodes and element lengths. The effect of semi-rigid connections could be included in the second-order plastic hinge analysis using the zero length rotational spring element approach. The elastic unloading and strain-hardening effects are not included in analysis.

The efficiency, accuracy and robustness of the analytic procedure and the computer program developed here has been evaluated using several benchmark problems analysed previously by other researchers using independent analytical, numerical and finite element solutions. The obtained results highlight the following key remarks:

- *Regarding the stiffness matrix coefficients evaluation for tapered elements in geometric nonlinear analysis*

In order to prove the stability of the proposed method, the power series based approach, extensive convergence studies have been made for circular and rectangular tapered beam that have been studied by other researchers of (Al-Sarraf, 1979), (Al-Sadder, 2004). It has been concluded that the values of the element stiffness coefficients predicted by the proposed element formulation are in very close agreement with those published in (Al-Sadder, 2004), it can be observed that the solutions by the present method converge very fast, usually 20-30 terms are sufficient to achieve reasonable accuracy even for large values of axial force parameter. It is worth noting that the high values assumed for the axially force in these computational examples are considered here and in (Al-Sadder, 2004) to assess the numerical stability of the developed methods. It is mentioned that for low values of axial force significant fewer terms are required in the present analysis as compared with that in (Al-Sadder, 2004). Such a result could be attributed to the fact that in our formulation power series solution is involved only in the computation of the second-order bending moment whereas in (Al-Sadder, 2004) the general solution of the displacement is obtained by applying the power series approach. For small values of axial force the analysis is closer to the first order theory and hence the flexibility based model proposed here performs better than displacement-based models used in (Al-Sadder, 2004). For practical situations, the proposed method requires significantly less terms in the power series to achieve reasonable accuracy.

Summarizing the conducted studies, it can be noticed that the integration method used for flexibility coefficients evaluation does not significantly influence the numbers of terms used in polynomial function, in order to ensure convergence. On the other hand, Gauss quadratures have faster convergence than Simpson 3/8 rule, requiring a minimum 3 integration points, compared to 6 integration points.

- *Regarding geometrically nonlinear analysis of planar steel frames with tapered members or individual tapered elements*

If *practical proposed method* is used for axial critical load evaluation, it is recommended that end section height ratio to be less than 0.6; otherwise errors higher than 8% have been reported. In this case, it is recommended the *power series approach* to be used which has proven to be stable and leads to results with higher precision.

In order to verify the efficiency and accuracy of the proposed approach, the power series approach, other studies have been conducted, that implies dividing the tapered element into several segments with constant section and it has been concluded that, generally, takes about 100 segments to achieve the desired accuracy.

If power series approach is used, for axial critical load evaluation, it can be seen the behaviour of the tapered beam predicted by the proposed approach is in very close agreement with the one developed in (Li & Li, 2002). Furthermore a small number of terms are required in the proposed approach to achieve convergence. It can also be concluded that in this case shear deformation influences the behaviour of tapered beam, the stiffness of the beam is reduced significantly due to shear deformation.

As it can be seen the proposed element formulation is able to accurately predict the geometrically nonlinear response of tapered columns, with and without considering the initial geometric imperfections, the load-deflection curves obtained with the proposed approach are in very close agreement in most cases with those published in (Raftoyiannis & Ermopoulos, 2005), (Li & Li, 2002), (Hadidi, Azar, & Marand, 2014), (2014). A slightly underestimation of the displacements may be observed in the proposed approach for a very high values of the applied load factors when only seven series terms have been used for solving the differential equilibrium equation. However, as noted in (Raftoyiannis & Ermopoulos, 2005) such situations will never reach in practice due to its material and geometrical characteristics of the columns, but instead material failure will occur for much smaller values of the applied compressive axial load N. It can be concluded that for axial loads bellow the ultimate load factor the load-deflection curves are in very close agreement with those obtained analytically in (Raftoyiannis & Ermopoulos, 2005) even for the situation when only six series terms are used. This example proves that usually for practical applications only few series terms are required in the proposed approach to obtain good accuracy.

Studies for geometrically nonlinear analysis of tapered beams with double variation have, also, been conducted and the obtained results agree very well with those obtained in finite element program Abaqus, but with much less computational effort. For modelling the tapered bar S4r shell elements with 10mm have been used in Abaqus and 2 nodes linear elements with 10 integration Gauss-Lobatto points each in EPASS.

The stability of a steel portal frames composed of non-prismatic members of tapered I-section studied by other researchers (Karabalis & Beskos, 1983), (Rezaiee-Pajand, Shahabian, & Bambaeechee, 2015), (Saffari, Rahgozar, & Jahanshahi, 2008), (Valipour & Bradford, 2012) has, also, been investigated. The obtained values prove that the proposed approach gives accurate results despite the fact that only one element has been used to model the tapered bars and a small number of terms have been considered in the power series.

- *Regarding elasto-plastic nonlinear analysis of elements or planar steel frames with prismatic or tapered members*

If classic plastic hinge approach with instant and punctual formation is used, an excellent correlation between the results obtained with the developed application, EPASS, and those mentioned in the literature has been reported (Ziemian, 1992), (Chiorean C. G., 2006). Spread of plasticity simulation has been considered by using tangent modulus Et relation proposed by Zubydan (2010). It should be mentioned that using a singer elements per member, the developed application, EPASS, gives results with satisfactory precision, even when including residual tensions or not, compared with fiber based analysis (Vogel, 1985), (Ziemian, 1992), (Chiorean C. G., 2006). It has been observed that including residual tensions in the analysis causes a more „smooth" stiffness degradation of the elements, but does not significantly influence the ultimate loading factor for structures with a high degree of statical indeterminacy. Also, it has been noted the influence of plastic interaction curse on the ultimate bearing capacity. Thus, for the performed analysis, Orbison plastic interaction curve overestimates ultimate loading factor by approximately 3.5% compared to the one proposed in the American AISC-LRFD code. However, once a plastic hinge is fully formed, the plastic interaction curve is not "violated", regardless of the one considered in the analysis.

The importance of including distributed lateral loads acting along the member length in the analysis model is well known, in daily structural design. Hence, this type of loads can be directly input into the analysis without the need of dividing the member into several elements leading to a consistency in the linear and the nonlinear structural models. The method, also, ensures that the plastic bending moment is nowhere exceeded along the member length once a plastic hinge develops without the need to divide the member into two elements and applying the plastic flow rules at the element ends as in (Orbison, McGuire, & Abel, 1982) or to make additional operations of static condensation as in (Liu, Liu, & Chan, 2014).

The initial geometric effects do not significantly influence the ultimate loading factor, for structures with high degree of statical indeterminacy, but in a geometrically nonlinear analysis these effects have a great impact on the deformability capacity of the elements.

To verify the performance and efficiency of the proposed method in capturing second-order effects, plastic behaviour and also the effects of initial geometric imperfections over ultimate strength capacity and deformability of initially- curved tapered steel elements, several sets of nonlinear elasto-plastic analyses have been conducted for axially compressed. It is worth noting that the control cross-sections, used to detect the plastic hinge development inside the element length are located accordingly with the assumed numerical integration scheme. Hence, the position of the integration points dominates the accuracy of detection of the interior plastic hinge development, usually the results are more accurate as the number of evaluation points increases. A sensitivity study has been performed concerning this issue and it can be seen a minimum number of eight integration points are sufficient to obtain reasonable accuracy compared to the ones obtained with finite element program Abaqus, and also by increasing the number of the integration points more accurate solutions are achieved. In this case, also, variation of the bending moment at the plastic hinge locations shows that when the plastic hinge is formed the plastic interaction requirement is fully accomplished.

The effects of semi-rigid connections with either linear or nonlinear behaviour have been also investigated. As it can be seen in the comparative load displacement curves of rigid and semi-rigid connections (with linear or nonlinear behaviour), the effects of semi-rigid connections are very important and must be considered in a valuable advanced analysis method.

Personal contributions

The main endeavour of the current paper is the development of a coherent nonlinear static analysis method that has been implemented in a computer program for the advanced nonlinear static analysis of steel frames composed of tapered elements. The main features, that give value to the proposed analytical formulation and the developed program, compared to other methods and software applications for nonlinear static analysis of frame steel structures composed of tapered elements, is that, unlike other finite element based methods that need a refined mesh in order to achieve the desired accuracy, the proposed method uses only one element per physical member. This approach leads to reduced number of degrees of freedom, the same used in linear analysis of structures. Both the reduced computational effort and the amount of information that the developed program provide make it a practical tool to structural engineers for the analysis of steel frames composed of tapered elements and semi-rigid connections.

Considering all the above, we believe that the following aspects are major contributions to the current knowledge in this field:

- The **development of a tapered element** able to capture the combined effects of second-order geometrical nonlinearities, material nonlinearities, shears deformation, initial geometrical and mechanical imperfections and semi-rigid connections. The equilibrium differential equation of the tapered element with the bending moment as main unknown is established, and it is successfully solved by utilizing **power series** approach. **Maxwell-Mohr method** is then used to compute the incremental force-displacement relationships of a general continuously non-prismatic Timoshenko-Euler beam-column element. The proposed governing equation and the resulted element stiffness matrix and equivalent nodal load vector are of general forms, they can be applied to any variation of sectional shape along member length.

- The calibration **a practical procedure for the evaluation of the stiffness matrix and equivalent nodal forces vector for tapered members**. The practical character of the proposed approach consists in integrating the bending moment expression as for prismatic element with an equivalent flexibility stiffness when using Maxwell-Mohr method in order to compute the force-displacement relationship at the element level, highlighting in this way the flexibility matrix and equivalent nodal loads of the tapered element. A series of calibration studies have been conducted that leads to the evaluation of the compression coefficient that takes into account the end depth ratio.
  This procedure is effective in daily design of structures, and it is shown that can be applied in most practical cases; the errors introduced are, generally, below 5%.

- An **efficient computer program, EPASS**, has been developed in Matlab 7.11 for the nonlinear static analysis of 2D steel frames composed of tapered elements. The nonlinear inelastic analysis employed herein takes the advantage of using only one beam-column element per physical member simultaneously considering the effects of second-order geometrical nonlinearities, shear deformation and initial geometric imperfections featuring in this way the ability to be used for practical applications by combining modelling benefits, computational efficiency and reasonable accuracy. Distributed lateral loads acting along the member length can be directly input into the analysis without the need to divide the member into several elements leading to a consistency in the linear and the nonlinear structural models. A plastic hinge method is adopted to simulate member plasticity; plastic hinges are assumed to be lumped either at the ends of the beam-column element or along the element length. The method ensures also that the plastic bending moment is nowhere exceeded along the member length once a plastic hinge develops without the need to divide the member into two elements. In the present paper the concentrated plasticity model proposed in (Chiorean C. G., 2009) for the derivation of force-displacement relation is extended for non-prismatic elements in conjunction with consideration of second order effects and lateral loads in the bending moment expression and initial geometric imperfections. The effect of semi-rigid connections is included in the second-order plastic hinge analysis using the zero length rotational spring element approach. The accuracy of the computer program developed here has been extensively tested through comparisons with the established results available from the literature. Sensitivities studies concerning the main factors that affect the convergence and numerical stability of the proposed approach has been conducted in the paper, a suitable number of the power series terms and Gauss-Lobbato integration points used in the proposed method has been found. It can be concluded that the proposed formulation is highly accurate, the results predicted by the proposed approach correlate very well with the advanced finite element model and other results retrieved from the open literature but with computational efficiency, usually only one element per member is necessary to analyse which demonstrates the superior efficiency of the formulation for the cases examined herein.

Future research directions

We believe that the proposed method will open new horizons in the progress of this field. The main future research directions in the area of this study focus on the following:

- **The implementation of a numerical method with displacement control or arc-length control.** It is well known that for steel structures with slender elements, phenomena like "snap-back" or "snap-through" are very likely to happen that leads to stability failure and not to the loss of load bearing capacity. Also, such a method can trace the complete load-displacement curve, including post-critical behaviour of structures. Hence, an advanced analysis should be able to simulate this kind of behaviour.

- **The extension of the proposed mathematical formulation and the developer computer program for the nonlinear static analysis of spatial frame steel structures.** In order to simulate a more accurate behaviour of complex 3D steel structures, the interaction between the elements is extremely important and is should be taken into account. Otherwise, an incorrect estimation of the ultimate resistance capacity of the entire structural system can lead to disadvantaged solutions both economically and structurally.

- **Considering the effect of lateral-torsional buckling of steel members.** These types of failures are very common for steel girders and could significantly influence the global response of the structure. Conventional structural design methods are unable to capture rigorously this structural behaviour that is produced by the combined effects of geometrical nonlinearities and lateral torsional buckling phenomena.

- **Including of finite node real size in the analysis.** When using linear elements, a main drawback is that it does not take into account precisely the influence of real geometric size of the nodes and the overall response may be overestimated, which could lead to some higher costs; or may be underestimated, which could lead to a disastrous structural response. Also, a very important aspect is shear stiffness of the nodes, which, in conventional methods, is assumed to be infinite, and that could be, in some cases, in discord with the real behaviour (stiffness) of the steel structures.

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